Model comparison and parameter learning in inverse problems
Optimal strategy and convergent algorithm

Master internship offer

Possible application fields: Astronomy, medicine (scanner, tomography, MRI,...), non destructive evaluation, remote sensing,...

Scientific context: Inverse problem, reconstruction and restoration, deconvolution, Fourier synthesis, inverse Radon, super-resolution,...

Signal-image issues: Model comparison / selection, self-calibrated and self-adaptive deconvolution, unsupervised learning.

Involved tools: Hierarchical models, Bayesian strategy, stochastic sampling,...

Computing environment: PC, Matlab, Automatic Differentiation and Deep Learning Toolbox.

Location: Groupe Signal – Image, IMS (Université de Bordeaux – CNRS – BINP), Talence, France.

Supervisors: J.-F. GIOVANNELLI, Groupe Signal – Image, IMS. The work will be conducted in relation with colleagues in Heriot-Watt University (Edinburgh).

Duration: Five or six months starting in January or February 2024.

Doctoral study: The internship could open to a PhD thesis on similar subjects.

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Context — Over the last few years, the increase in the quantity of collected information has led to the need for processing important amount of large data sets. In this context, processing methods must be entirely automated and cannot demand for human intervention in order to tune parameters. This is far more crucial in inverse problems [1] for image restoration for three reasons: (1) methods are founded on regularization (due to ill-possedness) and then appeal for regularization parameter and (2) modern observation systems are complex and require models with various parameters and (3) complex image models include several types of hidden / latent variables. A multifold problem has then to be solved [2–5] from a unique observation (*i.e.,* unsupervised scheme): estimate regularisation parameters (self-adapted issue), instrument parameters (self-calibrated question) and latent variables (namely augmented problem). In addition to parameter estimation, a more advanced question deals with model selection (*e.g.,* select noise model or adequate instrument model within a set of candidate, infer the number of class in a segmentation problem, test an hypothesis regarding the value of a parameter,...). This question, regarding quantitative comparison and automatic selection of models, is a main open problem in data science and this is the core issue of the work.

Methodological framework — From the methodological standpoint, the investigations come within the framework of hierarchical models and Bayesian strategies [6, 7]. This framework has become a cornerstone tool in the field of statistical learning and specifically in inverse problems [1] since it allows to include numerous variables, possibly with complex interactions and to account for diverse sources of information (properties of unknown object, instrument model, noise and signal level,...) [8, 9]. Ultimately, the methods rely on optimal estimation / selection that are computed by guaranteed stochastic sampling of a posterior distribution.

Anticipated contribution — From a more formal standpoint, the model comparison will rely on probabilities that result from what is referred to as *evidences* that are marginal likelihoods. The latter are very difficult to compute and this issue will be a central part of the work. We will investigate methods that combine state-of-the-art for (i) Bayesian marginalization together with (ii) stochastic sampler. Regarding (i) we will first investigate the so called Chib [10–12] and harmonic expectation approaches [7, 13] to provide a thorough evidence calculations and sound numerical computations. As for (ii), we will first investigate Langevin and Hamilton [14–16] algorithms based on gradient and Hessian of the posterior to provide efficient proposal in a Metropolis-Hastings algorithm. Their practical implementation could rely on the automatic differentiation tools used as standard by the most recent deep neural network tools.

References

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