Parameter learning in tomography Optimal strategy and convergent algorithm

Master internship offer

Possible application fields: Medicine and biology, physics and astrophysics (*e.g.*, fluid mechanics, plasma), archaeology, non destructive evaluation...

Scientific context: Inverse problem, image reconstruction, inverse Radon, deconvolution, Fourier synthesis, Fourier slice theorem, super-resolution,...

Signal-image issues: Self-calibrated and self-adaptive inversion, unsupervised learning.

Involved tools: Penalties and constraints (deterministic), hierarchical models and Bayesian strategy (stochastic), optimization and sampling,...

Computing environment: PC, Matlab, Parallel Computing & Statistics and Machine Learning Toolboxes.

Location: Groupe Signal – Image, IMS (Université de Bordeaux – CNRS – BINP), Talence, France.

Supervisors: J.-F. GIOVANNELLI (IMS). Collaboration with Claire MICHELET (LP2IB) and Pascal DES-BARATS (LaBRI). The work could be in relation with colleagues in Heriot-Watt University (Edinburgh).

Duration: Five or six months starting in January or February 2024.

Doctoral study: The internship could open to a PhD thesis on similar subjects.

Contact: J.-F. GIOVANNELLI (Giova@IMS-Bordeaux.fr).

Context — A broad variety of measurement systems have become widespread in recent years, requiring processing tools adapted to important amounts of large data sets. In this context, processing methods must be entirely automated and cannot demand for human assistance in order to tune parameters. This is far more crucial in inverse problems [1] for image reconstruction (tomography, inverse Radon, scanner) for one main reason: methods are founded on regularization (required by ill-possedness *e.g.*, due to limited-angle) and then appeal for hyperparameters to balance a compromise between various types of information. Additionally, modern observation systems are complex and require models with various instrument parameters. A threefold problem has then to be solved [2-5]: from a unique observation (*i.e.*, unsupervised scheme) estimate parameters for object and noise (self-adapted issue) as well as instrument parameters (self-calibrated question), in addition to the image itself. This question, regarding parameters (object, noise, instrument) is a main open problem in data science and this is the core issue of the work.

Methodological framework — From a methodological point of view, the investigations planned for this internship falls into two categories [1, 6].

- 1. Deterministic framework: optimization [7,8] of criterion including penalties and constraints.
- 2. Stochastic framework: Bayesian strategies for hierarchical models [9] and stochastic sampling [10, 11].

These frameworks have become cornerstone tools in the field of inverse problems and especially tomography since they allow (a) to include various variables, possibly with complex interactions and (b) to account for diverse information: properties of unknown object (*e.g.*, edge-preservation and homogeneity, textures, positivity), instrument model, noise and signal level,... Ultimately, the second framework provides an optimal estimation function and also allows to include parameters and provides an idea of uncertainties.

Anticipated contribution — Regarding the second category, from a computational viewpoint, the work will rely on Markov chain Monte Carlo (MCMC) algorithms, stemming from the theory of Markov processes. Constructing efficient MCMC algorithms for high-dimensional problems is difficult, and this has stimulated a lot of research. We will first investigate Langevin and Hamilton [12–14] algorithms based on gradient and Hessian of the posterior to provide efficient proposal in a Metropolis-Hastings algorithm. We will include Fisher information to improve efficiency. The practical implementation could rely on the automatic differentiation tools used as standard by the most recent learning tools.

References

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