



Regularization methods for Radar Cross Section (RCS) imaging

2020-2021, 04/12/2020

Concept of Radar Cross Section (RCS) imaging

SPRITE : the proposed method for 3D RCS imaging

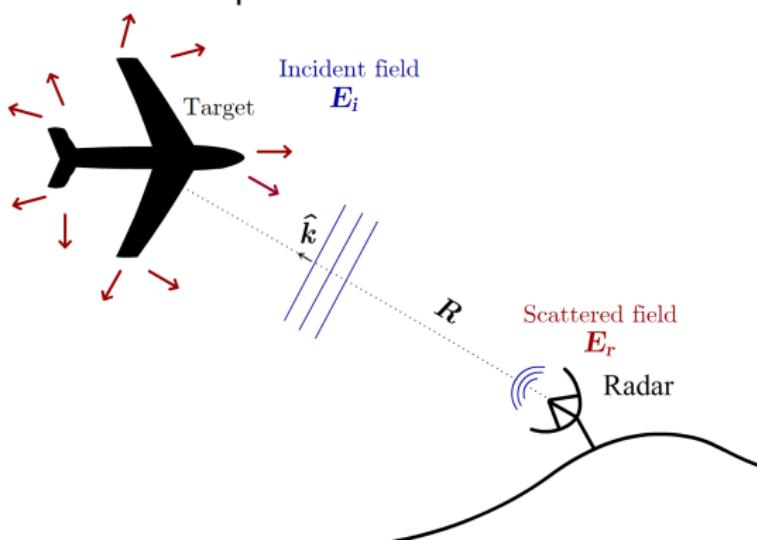
Conclusion

Concept of Radar Cross Section (RCS)

RCS → « measure of how detectable an object is by a radar »

Many factors influence the RCS :

- the material of which the target is made
- the geometry of the target
- the relative size of the target in relation to the wavelength
- the incident angle
- the polarization of transmitted and the received EM waves



$$\text{RCS} = 10 \log_{10} |\sigma|^2$$

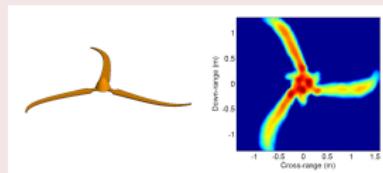
$$\sigma = 2\sqrt{\pi} R \frac{E_r}{E_i}$$

Typical RCS values

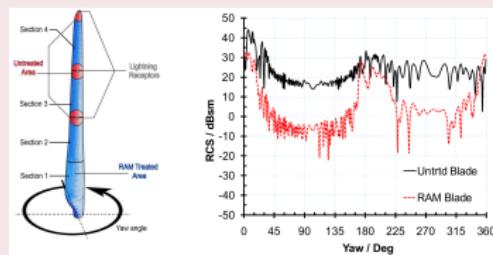
Targets	Typical RCS
 Navy cruiser	15000 m ² (83.5 dBm ²)
 B-52	125 m ² (41.9 dBm ²)
 Car	100 m ² (40 dBm ²)
 Bicycle	2 m ² (6 dBm ²)
 Human	1 m ² (0 dBm ²)
 Stealth aircraft	< 0.1 m ² (< -20 dBm ²)
 Bird	0.01 m ² (-40 dBm ²)
 Insect	0.00001 m ² (-100 dBm ²)

Radar cross-section analysis of wind turbine blades [RAS11]

Wind turbines interfere with radar systems due to their large RCS.



Possible mitigation measure : apply Radar Absorbing Materials.



[RAS11] RCS Analysis of Wind Turbine Blades with Radar Absorbing Materials, EuMA 2011, Rashid, L.S. & Brown, A.K.

Stealth targets → RCS reduction

Different scattering mechanisms

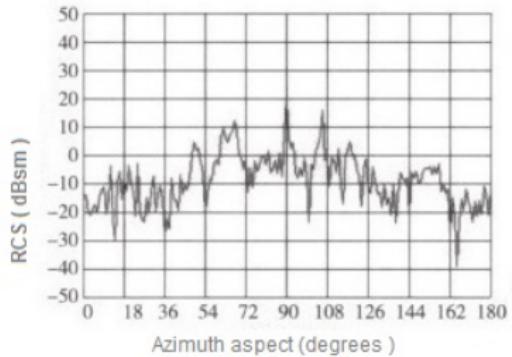


Stealth targets ⇒ RCS reduction by :
applying RAM
fine-tuning the design (e.g. reduce surface gap and edges).

Some examples of application (3)



F-35 RCS at 10 GHz , 20 degrees look up angle



Source : Lockheed Martin

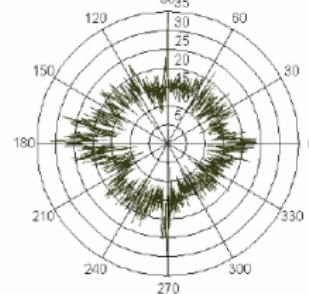
Some examples of application (3)

RCS graph of various vehicles at 94 GHz



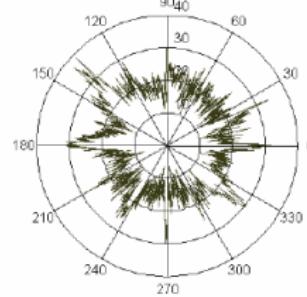
Bedford Truck

POLAR RADAR CROSS-SECTION DIAGRAM: BEDFORD



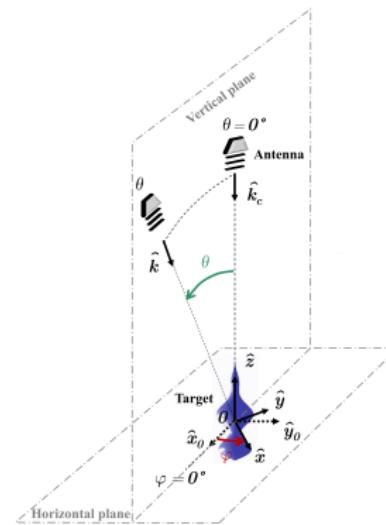
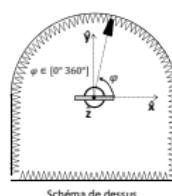
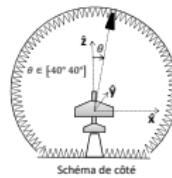
Armoured Personnel Carrier

POLAR RADAR CROSS-SECTION DIAGRAM: RATEL



Goal : form the map of the scatterers with an increased resolution.

Thematics : inverse problem, regularization, sparsity...



- Acquisition of M scattering coefficients for different angles and frequencies : $\sigma^{[m]} = \sigma(f^{[m]}, \theta^{[m]}, \varphi^{[m]}), \quad m = 0, \dots, M - 1$
- Merged into a vector : $\boldsymbol{\sigma} = [\sigma^{[0]}, \sigma^{[1]}, \dots, \sigma^{[M-1]}]^t$

$$\boldsymbol{\sigma} = \mathbf{H}\mathbf{a} + \mathbf{b}$$

- $\boldsymbol{\sigma}$: vector of observations.
- \mathbf{a} : vectorized RCS map.
- \mathbf{b} : noise vector.
- \mathbf{H} : model matrix.

$$\hookrightarrow \mathbf{H} = \alpha\sqrt{N} \Delta_\Psi S F_{3D} \Delta_\Phi$$

- ▷ α : complex coefficient .
- ▷ $N = N_x N_y N_z$: number of voxels of the map.
- ▷ S : binary selection matrix.
- ▷ F_{3D} : 3D discrete Fourier Transform matrix.
- ▷ Δ_Ψ : diagonal phase shifting matrix in the k-space.
- ▷ Δ_Φ : diagonal phase shifting matrix in the spatial domain.

Inverse problem

Estimate the map a from the observations σ .

Difficulties

- H is rank deficient \Rightarrow the matrix is not invertible.
- Noisy data.
- 3D high dimensional problem.

Regularization

Need to introduce additional information on a to compensate for the missing information on σ .

Regularization methods (see lectures and practical works)

Missing information \Rightarrow Need to add prior information = Regularization

\hookrightarrow penalty

- linked term : linear combinations, inter-pixel differences (Dx)...
 - $\triangleright \|Dx\|_2 \rightarrow$ smooth restoration
 - $\triangleright \varphi_H(Dx) \rightarrow$ piecewise smooth restoration
 - $\triangleright \|Dx\|_1 \rightarrow$ piecewise constant restoration ...
- separable term
 - $\triangleright \|x\|_2 \rightarrow$ low energy restoration
 - $\triangleright \|x\|_1 \rightarrow$ sparse restoration ...

\hookrightarrow constraints...

Minimum Norm Least Squares

$$\hat{\mathbf{a}} = \underset{\mathbf{a}}{\operatorname{argmin}} \left\{ \begin{array}{l} \|\mathbf{a}\|_2^2 \\ \text{s.t. } \boldsymbol{\sigma} = \mathbf{H}\mathbf{a} \end{array} \right.$$
$$= \mathbf{H}^\dagger (\mathbf{H}\mathbf{H}^\dagger)^{-1} \boldsymbol{\sigma}$$

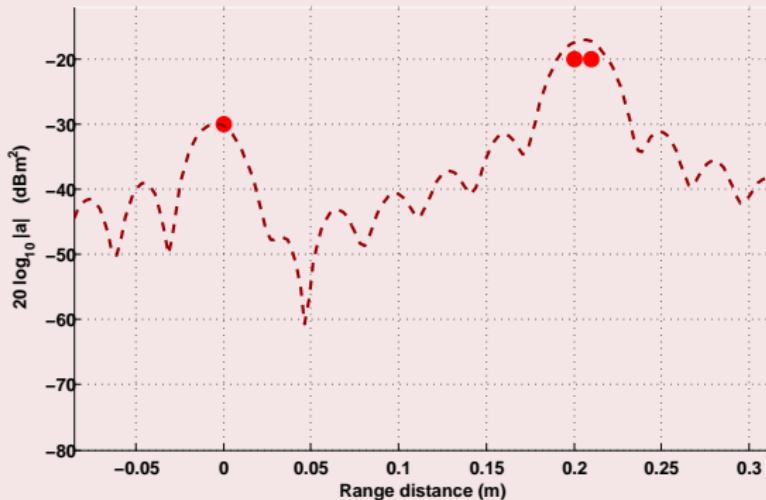
After regridding :

$$\hat{\mathbf{a}} = \alpha^* \frac{1}{\sqrt{N}} \boldsymbol{\Delta}_\Phi^* \mathbf{F}_{3D}^\dagger \mathbf{S}^t \boldsymbol{\Delta}_\Psi^* \check{\boldsymbol{\sigma}}$$



- Large main lobes.
- High sidelobes level.

IFFT with zero padding

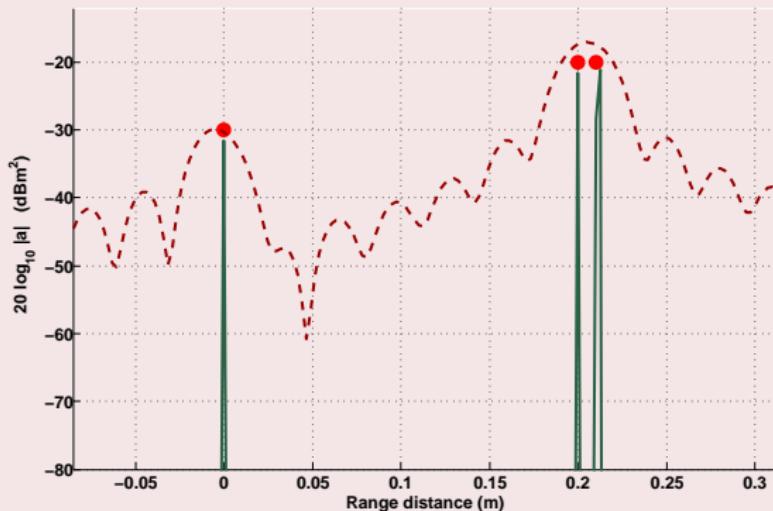


- Finite bandwidth of the radar → sinc defocusing Point Spread Function around scatterers
- Weak scatterers may be hidden by high sidelobe level
 - Resolution too limited for precise RCS analysis.

Sparse approaches

Increase the resolution with *prior* information on the solution

- A small number of scatterers \rightsquigarrow sparsity-driven approach.
- Limited electromagnetic extension \rightsquigarrow support constraint.



How to promote sparsity ?

- L_0 pseudo norm

▷ $\|\mathbf{a}\|_0 := \text{Card}(k | a_k \neq 0)$ (ie. number of nonzero components of \mathbf{a})

$$\hookrightarrow \text{penalized form } \hat{\mathbf{a}} = \underset{\mathbf{a} \in \mathbb{C}^K}{\operatorname{argmin}} \|\boldsymbol{\sigma} - \mathbf{H}\mathbf{a}\|_2^2 + \mu \|\mathbf{a}\|_0$$

$$\hookrightarrow \text{constrained form } \hat{\mathbf{a}} = \begin{cases} \underset{\mathbf{a} \in \mathbb{C}^K}{\operatorname{argmin}} \|\mathbf{a}\|_0 \\ \text{s.t. } \boldsymbol{\sigma} = \mathbf{H}\mathbf{a} \end{cases}$$

L_0 norm is not convex \Rightarrow hard combinatorial optimization problem
(idem for L_p norms with $p < 1$)

How to promote sparsity ?

- L_1 norm (relaxation of the L_0 pseudo norm)

▷ $\|a\|_1 := \sum_k |a_k|$

again various forms (constrained or penalized)

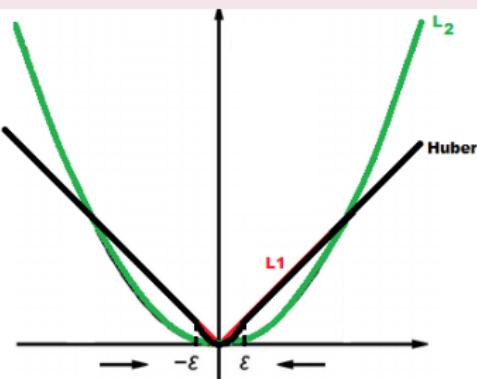
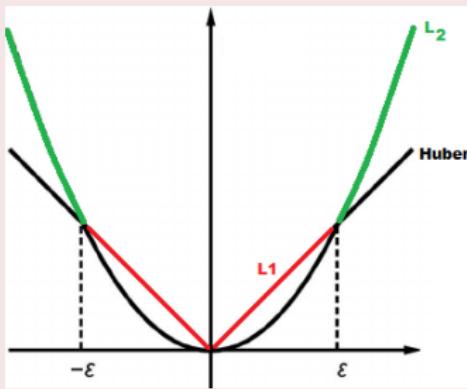
convex !...

...but not differentiable

(see exercise "Quadratic penalty versus Absolute penalty")

How to promote sparsity ?

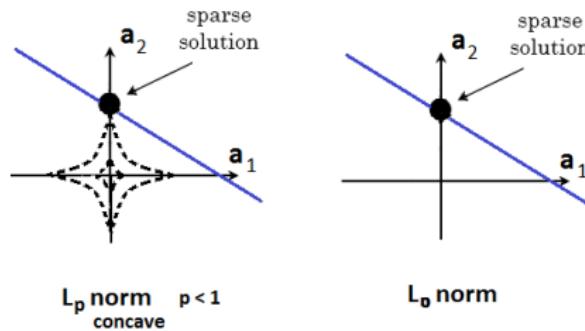
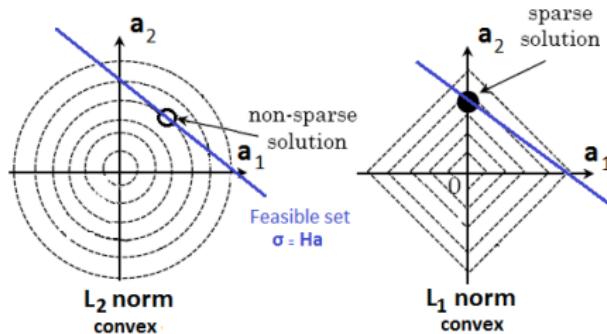
- $L_2 - L_1$ potential (e.g. Huber potential) with a low threshold $\rightsquigarrow L_1$



(refer to lecture and practical work)

Sparse approaches

For instance, let's consider $a = [a_1, a_2]^t$ is a 2D vector.



Prior information**Associated penalty****(P1)**projection of a onto \hat{z} is sparse

$$\|Pa\|_1$$

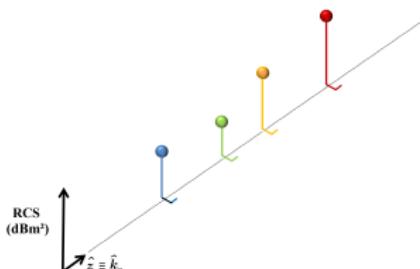
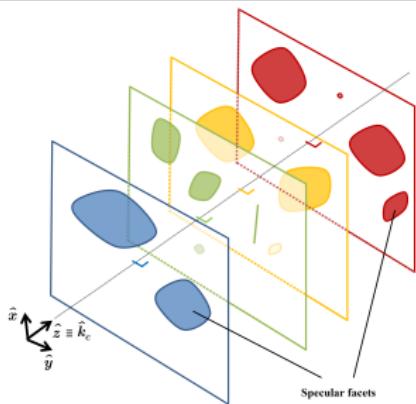
(P2)

specular facets are compact

(P3)

the map is constant all over each specular facets

$$\|D_x a\|_1 \text{ and } \|D_y a\|_1$$



Prior information	Associated penalty
(P1) projection of a onto \hat{z} is sparse	$\ Pa\ _1$
(P2) specular facets are compact	
(P3) the map is constant all over each specular facets	$\ D_x a\ _1$ and $\ D_y a\ _1$
(P4) low energy	$\ a\ _2^2$

$$\mathcal{J}(a) = \frac{1}{2} \|\sigma - Ha\|_2^2 + \mu \|Pa\|_1 + \lambda \|D_x a\|_1 + \lambda \|D_y a\|_1 + \frac{\nu}{2} \|a\|_2^2$$

(P5) EM extent of the target lies in a finite spatial support \leadsto support constraints.

$$\hat{\mathbf{a}} = \underset{\mathbf{a} \in \mathbb{C}^N}{\operatorname{argmin}} \left\{ \begin{array}{l} \frac{1}{2} \|\boldsymbol{\sigma} - \mathbf{H}\mathbf{a}\|_2^2 + \mu \|\mathbf{P}\mathbf{a}\|_1 + \lambda \|\mathbf{D}_x \mathbf{a}\|_1 + \lambda \|\mathbf{D}_y \mathbf{a}\|_1 + \frac{\nu}{2} \|\mathbf{a}\|_2^2 \\ \text{s.c. } \mathbf{a} \in \mathcal{C} \end{array} \right.$$

Minimization difficulties

- Non differentiable criterion (due to ℓ_1 norms).
- High dimensional problem (> 1 million unknowns).

Alternating Direction Method of Multipliers (ADMM)

- Convex optimization algorithm.
- Lies on an augmented Lagrangian method.
- Very suitable for our criterion.
- Guaranteed convergence in our case [BOY10].

Re-writing of the solution with an equivalent ADMM form

$$\hat{\mathbf{a}} = \underset{\mathbf{a}}{\operatorname{argmin}} \left\{ \begin{array}{l} \frac{1}{2} \|\boldsymbol{\sigma} - \mathbf{H}\mathbf{a}\|_2^2 + \mu \|\mathbf{P}\mathbf{a}\|_1 + \lambda \|\mathbf{D}_x\mathbf{a}\|_1 + \lambda \|\mathbf{D}_y\mathbf{a}\|_1 + \frac{\nu}{2} \|\mathbf{a}\|_2^2 \\ \text{s.c. } \mathbf{a} \in \mathcal{C} \end{array} \right.$$

$$\hat{\mathbf{a}} = \underset{\mathbf{a}, \mathbf{v}}{\operatorname{argmin}} \left\{ \begin{array}{l} \frac{1}{2} \|\boldsymbol{\sigma} - \mathbf{H}\mathbf{a}\|_2^2 + \mu \|\mathbf{v}_P\|_1 + \lambda \|\mathbf{v}_x\|_1 + \lambda \|\mathbf{v}_y\|_1 + \frac{\nu}{2} \|\mathbf{a}\|_2^2 + \mathcal{I}_C(\mathbf{v}_C) \\ \text{s.t. } \left\{ \begin{array}{l} \mathbf{v}_P = \mathbf{P}\mathbf{a} \\ \mathbf{v}_x = \mathbf{D}_x\mathbf{a} \\ \mathbf{v}_y = \mathbf{D}_y\mathbf{a} \\ \mathbf{v}_C = \mathbf{a} \end{array} \right. \end{array} \right.$$

- $\mathbf{v} = [\mathbf{v}_P, \mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_C]$: auxiliary variables.

Minimization of the criterion with the ADMM

$$\hat{\mathbf{a}} = \underset{\mathbf{a}, \mathbf{v}}{\operatorname{argmin}} \left\{ \begin{array}{l} \frac{1}{2} \|\boldsymbol{\sigma} - \mathbf{H}\mathbf{a}\|_2^2 + \mu \|\mathbf{v}_P\|_1 + \lambda \|\mathbf{v}_x\|_1 + \lambda \|\mathbf{v}_y\|_1 + \frac{\nu}{2} \|\mathbf{a}\|_2^2 + \mathcal{I}_C(\mathbf{v}_C) \\ \text{s.t. } \left\{ \begin{array}{l} \mathbf{v}_P = \mathbf{P}\mathbf{a} \\ \mathbf{v}_x = \mathbf{D}_x\mathbf{a} \\ \mathbf{v}_y = \mathbf{D}_y\mathbf{a} \\ \mathbf{v}_C = \mathbf{a} \end{array} \right. \end{array} \right\}$$

Normalized augmented Lagrangian :

$$\begin{aligned} \mathcal{L}_\rho(\mathbf{a}, \mathbf{v}, \mathbf{u}) &= \frac{1}{2} \|\boldsymbol{\sigma} - \mathbf{H}\mathbf{a}\|_2^2 + \mu \|\mathbf{v}_P\|_1 + \lambda \|\mathbf{v}_x\|_1 + \lambda \|\mathbf{v}_y\|_1 + \frac{\nu}{2} \|\mathbf{a}\|_2^2 + \mathcal{I}_C(\mathbf{v}_C) \\ &\quad + \frac{\rho_P}{2} \|\mathbf{P}\mathbf{a} - \mathbf{v}_P + \mathbf{u}_P\|_2^2 - \frac{\rho_P}{2} \|\mathbf{u}_P\|_2^2 + \frac{\rho_D}{2} \|\mathbf{D}_x\mathbf{a} - \mathbf{v}_x + \mathbf{u}_x\|_2^2 - \frac{\rho_D}{2} \|\mathbf{u}_x\|_2^2 \\ &\quad + \frac{\rho_D}{2} \|\mathbf{D}_y\mathbf{a} - \mathbf{v}_y + \mathbf{u}_y\|_2^2 - \frac{\rho_D}{2} \|\mathbf{u}_y\|_2^2 + \frac{\rho_C}{2} \|\mathbf{a} - \mathbf{v}_C + \mathbf{u}_C\|_2^2 - \frac{\rho_C}{2} \|\mathbf{u}_C\|_2^2 \end{aligned}$$

- $\mathbf{u} = \left[\frac{\gamma_P}{\rho_P}, \frac{\gamma_x}{\rho_D}, \frac{\gamma_y}{\rho_D}, \frac{\gamma_C}{\rho_C} \right] = [\mathbf{u}_P, \mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_C]$: normalized Lagrange multipliers.
- $\rho = [\rho_P, \rho_D, \rho_C]$: strictly positive penalty parameters.

Initialize $v^{(0)}$ et $u^{(0)}$.

Repeat until convergence

$$\mathbf{a}^{(k+1)} = \underset{\mathbf{a}}{\operatorname{argmin}} \mathcal{L}_\rho(\mathbf{a}, \mathbf{v}^{(k)}, \mathbf{u}^{(k)})$$

$$\mathbf{v}_P^{(k+1)} = \underset{\mathbf{v}_P}{\operatorname{argmin}} \mathcal{L}_\rho(\mathbf{a}^{(k+1)}, \mathbf{v}_P, \mathbf{v}_x^{(k)}, \mathbf{v}_y^{(k)}, \mathbf{v}_C^{(k)}, \mathbf{u}^{(k)})$$

$$\mathbf{v}_x^{(k+1)} = \underset{\mathbf{v}_x}{\operatorname{argmin}} \mathcal{L}_\rho(\mathbf{a}^{(k+1)}, \mathbf{v}_P^{(k+1)}, \mathbf{v}_x, \mathbf{v}_y^{(k)}, \mathbf{v}_C^{(k)}, \mathbf{u}^{(k)})$$

$$\mathbf{v}_y^{(k+1)} = \underset{\mathbf{v}_y}{\operatorname{argmin}} \mathcal{L}_\rho(\mathbf{a}^{(k+1)}, \mathbf{v}_P^{(k+1)}, \mathbf{v}_x^{(k+1)}, \mathbf{v}_y, \mathbf{v}_C^{(k)}, \mathbf{u}^{(k)})$$

$$\mathbf{v}_C^{(k+1)} = \underset{\mathbf{v}_C}{\operatorname{argmin}} \mathcal{L}_\rho(\mathbf{a}^{(k+1)}, \mathbf{v}_P^{(k+1)}, \mathbf{v}_x^{(k+1)}, \mathbf{v}_y^{(k+1)}, \mathbf{v}_C, \mathbf{u}^{(k)})$$

$$\mathbf{u}_P^{(k+1)} = \mathbf{u}_P^{(k)} + \mathbf{P} \mathbf{a}^{(k+1)} - \mathbf{v}_P^{(k+1)}$$

$$\mathbf{u}_x^{(k+1)} = \mathbf{u}_x^{(k)} + \mathbf{D}_x \mathbf{a}^{(k+1)} - \mathbf{v}_x^{(k+1)}$$

$$\mathbf{u}_y^{(k+1)} = \mathbf{u}_y^{(k)} + \mathbf{D}_y \mathbf{a}^{(k+1)} - \mathbf{v}_y^{(k+1)}$$

$$\mathbf{u}_C^{(k+1)} = \mathbf{u}_C^{(k)} + \mathbf{a}^{(k+1)} - \mathbf{v}_C^{(k+1)}$$

$\mathbf{v}_P, \mathbf{v}_x, \mathbf{v}_y$ and \mathbf{v}_C & $\mathbf{u}_P, \mathbf{u}_x, \mathbf{u}_y$ and \mathbf{u}_C updates

Separable updates (can be done in parallel), direct and very simple !

a update : very fast in the Fourier domain

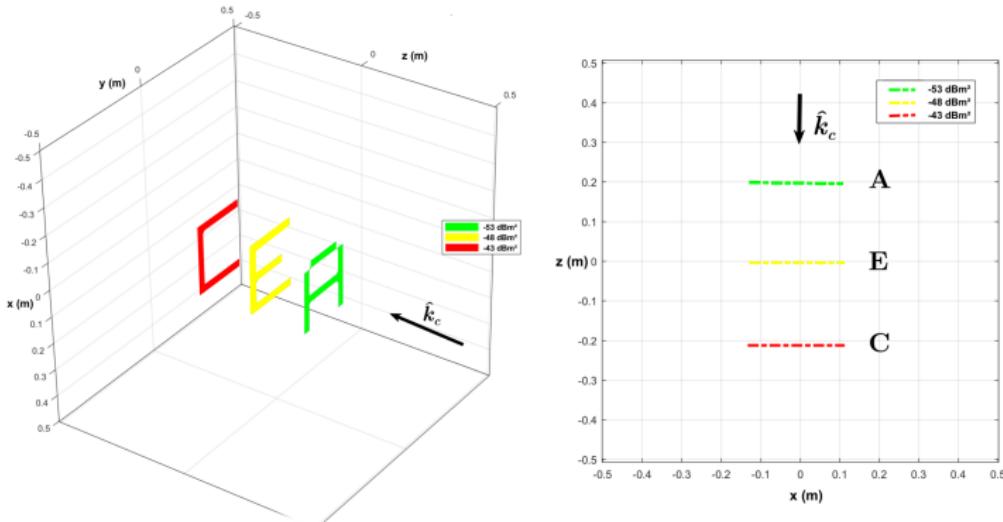
$$\mathbf{a}^{(k+1)} = \alpha N \mathbf{F}_{3D}^{-1} \boldsymbol{\Lambda}_G^{-1} \mathbf{F}_{3D} (\boldsymbol{\Delta}_\Phi^* \mathbf{F}_{3D}^\dagger \mathbf{S}^t \boldsymbol{\Delta}_\Psi^* \check{\boldsymbol{\sigma}} + \mathbf{t}^{(k)})$$

- $\boldsymbol{\Lambda}_G = \mathbf{F}_{3D} (\mathbf{H}^\dagger \mathbf{H} + \rho_P \mathbf{P}^\dagger \mathbf{P} + \rho_D (\mathbf{D}_x^\dagger \mathbf{D}_x + \mathbf{D}_y^\dagger \mathbf{D}_y) + (\nu + \rho_C) \mathbf{I}_N) \mathbf{F}_{3D}^\dagger$
Diagonal invertible matrix. Computed once before starting algorithm.
- $\boldsymbol{\Delta}_\Phi^* \mathbf{F}_{3D}^\dagger \mathbf{S}^t \boldsymbol{\Delta}_\Psi^* \check{\boldsymbol{\sigma}}$: PFA map. Also computed once before starting algorithm.
- $\mathbf{t}^{(k)} = \rho_P \mathbf{P}^\dagger (\mathbf{v}_P^{(k)} - \mathbf{u}_P^{(k)}) + \rho_D [\mathbf{D}_x^\dagger (\mathbf{v}_x^{(k)} - \mathbf{u}_x^{(k)}) + \mathbf{D}_y^\dagger (\mathbf{v}_y^{(k)} - \mathbf{u}_y^{(k)})] + \rho_C (\mathbf{v}_C^{(k)} - \mathbf{u}_C^{(k)})$

Very simple and fast update by using 3D FFT !

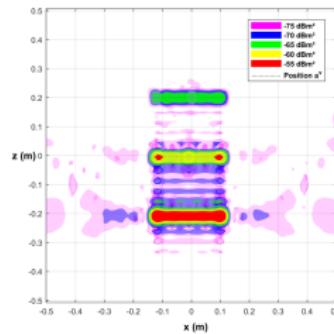
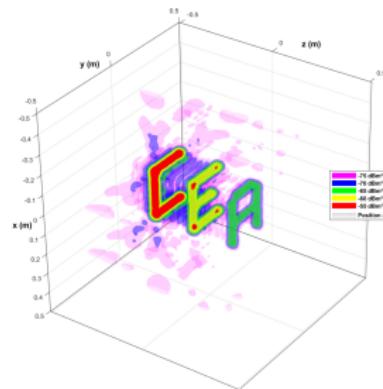
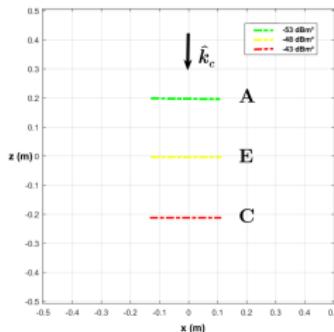
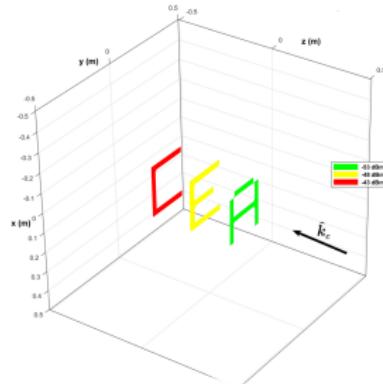
Validation on simulated data : CEA map

Facet	C	E	A
Longitudinal position (m)	-0,21	0	0,20
Magnitude (dB)	-40	-45	-50
Phase (rad)	$\pi/4$	$\pi/2$	$\pi/3$

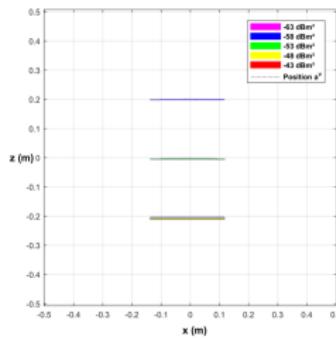
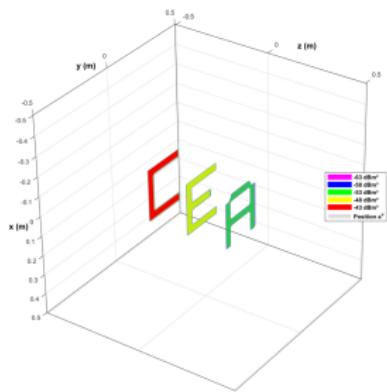
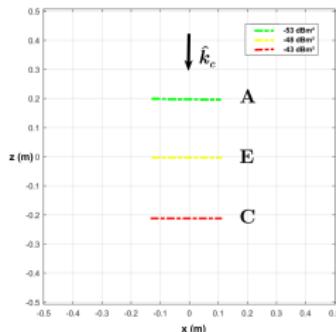
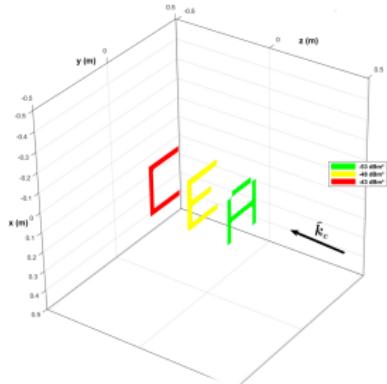


- $f = [8 \text{ GHz} : 5 \text{ MHz} : 12 \text{ GHz}]$, $\theta = [-15^\circ : 1^\circ : 15^\circ]$, $\varphi = [-15^\circ : 1^\circ : 15^\circ]$
- + white Gaussian noise with SNR = -7,7 dB.

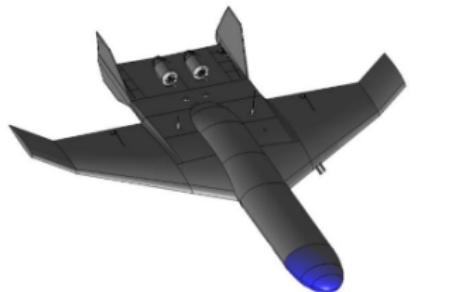
Validation on simulated data : PFA



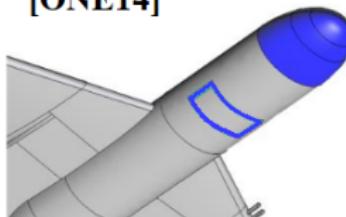
Validation on simulated data : SPRITE



SPRITE parameters : $\mu = 10$, $\lambda = 100$, $\nu = 150$



[ONE14]



- HH polarization
- $f = [2.5 \text{ GHz} : 30 \text{ MHz} : 4 \text{ GHz}]$, $\theta = [-20^\circ : 1^\circ : 20^\circ]$, $\varphi = [-20^\circ : 1^\circ : 20^\circ]$
- + white Gaussian noise bruit with SNR = 15 dB.

RCS analysis : drone

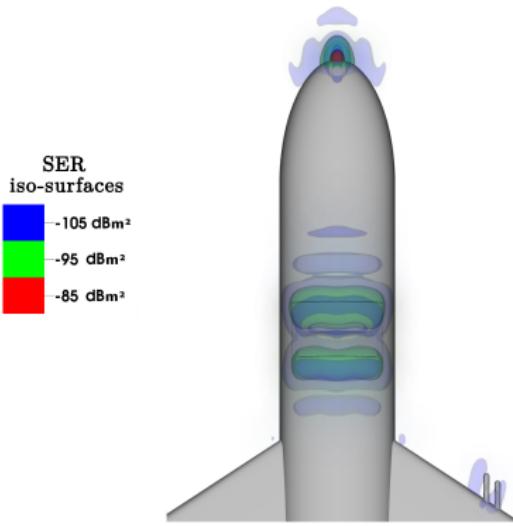
PFA



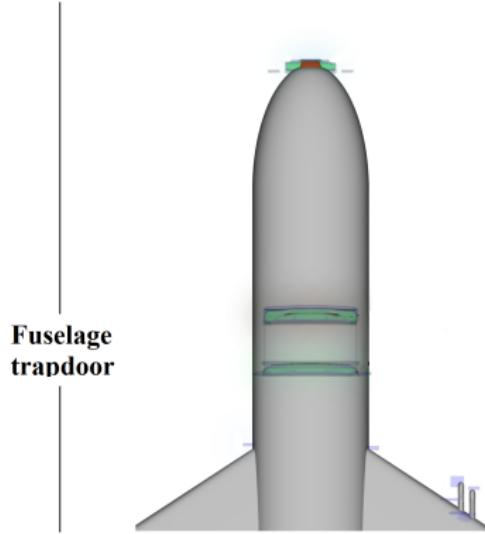
SPRITE



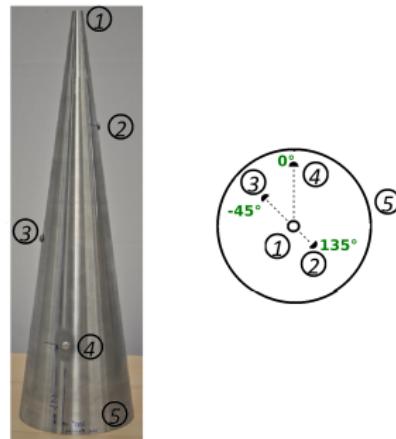
SPRITE parameters : $\mu = 10$, $\lambda = 120$, $\nu = 100$



PFA

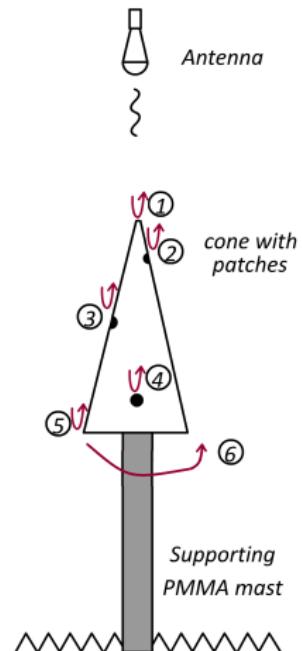
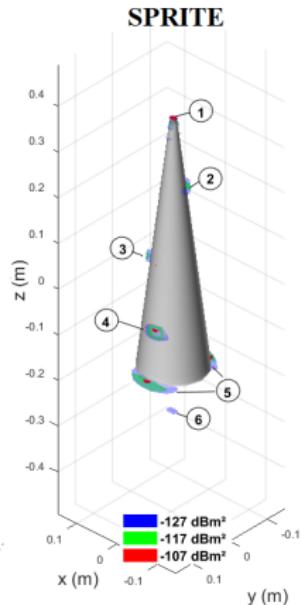
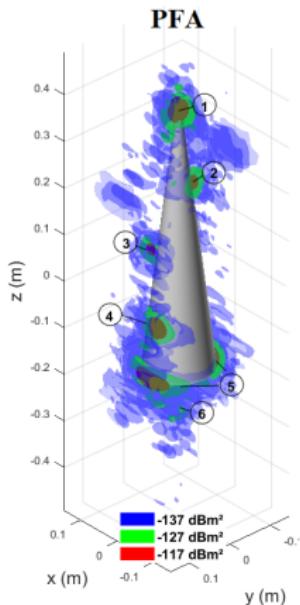


SPRITE



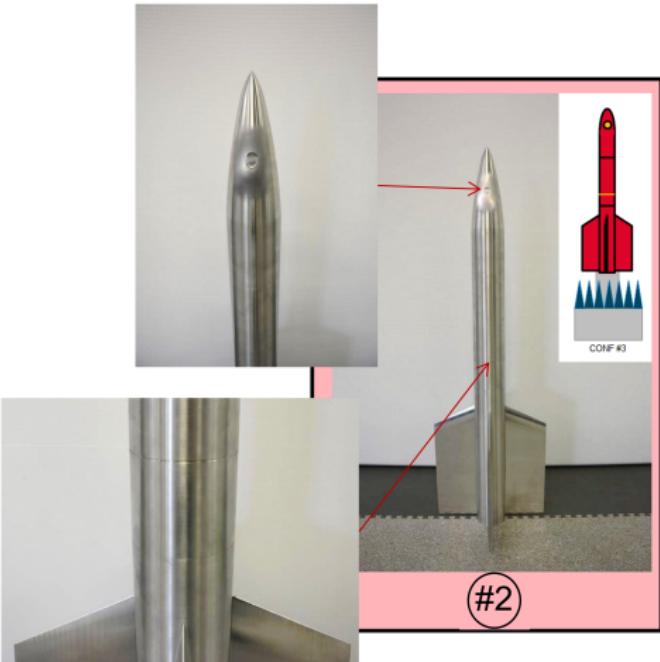
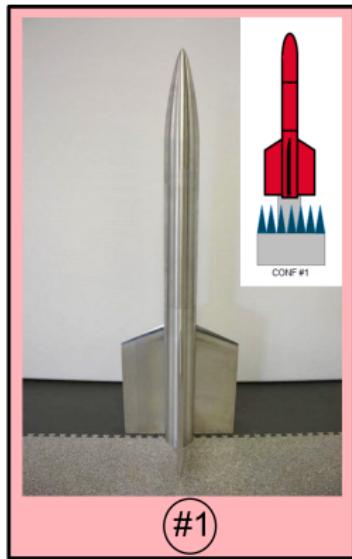
- HH polarization
- $f = [8 \text{ GHz} : 3,9 \text{ MHz} : 12 \text{ GHz}]$
- $\theta = [-20^\circ : 1^\circ : 20^\circ]$, $\varphi = [-20^\circ : 4^\circ : 20^\circ]$

RCS analysis : cone



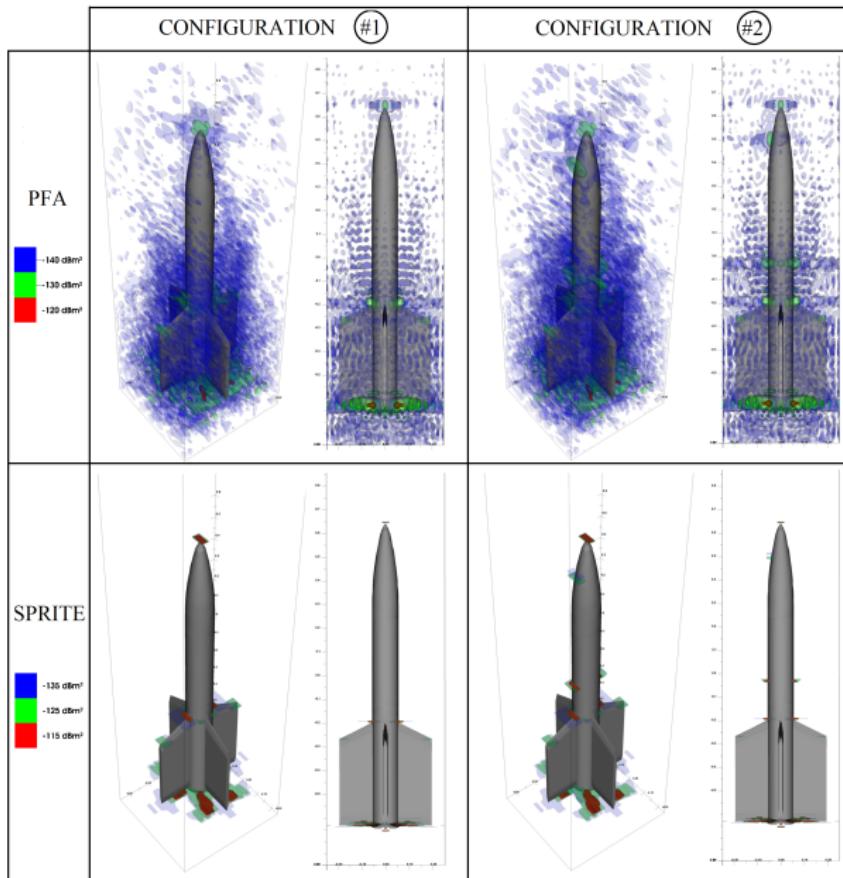
SPRITE parameters : $\mu = 5$, $\lambda = 10$, $\nu = 100$

RCS control : data measured with the Arche 3D



- VV polarization
- $f = [8 \text{ GHz} : 3,9 \text{ MHz} : 12 \text{ GHz}]$
- $\theta = [-20^\circ : 1^\circ : 20^\circ]$, $\varphi = [-20^\circ : 4^\circ : 20^\circ]$

RCS analysis : multi-stage launcher MX-14



SPRITE parameters : $\mu = 5$, $\lambda = 15$, $\nu = 50$

About inverse problem

- Inverse problem approaches can tackle a lot of engineering problems :
 - ▷ astronomy
 - ▷ medical imaging
 - ▷ RCS imaging...
- If the problem is ill-posed we need to add *prior* information = regularization

About my thesis work

- The sought solution is sparse and constrained.
- Sparsity can be achieved via L_1 norm minimization.
- The ADMM is an efficient algorithm to minimize such problems.
- The resolution is enhanced allowing a deeper characterization of the target scattering behavior.

Some publications

[BEN16] Méthode Semi-Quadratique et multiplicateurs de Lagrange pour l'imagerie RADAR, JIONC 2016, GDR ISIS/Ondes, *Benoudiba-Campanini & all*

[BEN17a] A New Regularization Method for Radar Cross Section Imaging, EuCAP 2017, *Benoudiba--Campanini, Minvielle, Massaloux, Giovannelli(*)*

[BEN17b] Régularisation parcimonieuse pour l'imagerie radar haute résolution 1D, JNM 2017, *Benoudiba--Campanini, Minvielle, Massaloux, Giovannelli(*)*

[BEN19] SPRITE : a New Sparse Approach for 3D High Resolution RCS Imaging, RADAR 2019, *Benoudiba--Campanini, Giovannelli, Minvielle, Massaloux (*)*

(*) «*Best papers award* »